2-6 EXAMPLES

1. Evaluate the following limits and justify your answers.

   (a) $\lim_{x\to-\infty} \frac{x+2}{2+x^2} \cdot \frac{1/x^2}{1/x^2} = \lim_{x\to-\infty} \frac{x^{-1}+2x^{-2}}{2x^{-2}+1} = \frac{0+0}{0+1} = 0$

   (b) $\lim_{x\to\infty} \frac{1-x^3}{x+4x^2} \cdot \frac{1/x^2}{1/x^2} = \lim_{x\to\infty} \frac{x^{-2}-x}{x^{-1}+4} = -\infty$

   (c) $\lim_{x\to\infty} \frac{3\sqrt{x}+1}{4\sqrt{x}-1} \cdot \frac{1/\sqrt{x}}{1/\sqrt{x}} = \lim_{x\to\infty} \frac{3+1/\sqrt{x}}{4-1/\sqrt{x}} = \frac{3+0}{4-0} = 3/4$

   (d) $\lim_{x\to-\infty} \frac{\sqrt{x+x^4}}{2+x^2} \cdot \frac{1/x^2}{1/x^2} = \lim_{x\to-\infty} \frac{\sqrt{x^{-3}+1}}{2x^{-2}+1} = 1$

(e) $\lim_{x\to\infty} [\ln(x^2+\sqrt{2}) - \ln(3x^2-x)] = \lim_{x\to\infty} \ln\left(\frac{x^2+\sqrt{2}}{3x^2-x}\right)$

$= \ln\left[\lim_{x\to\infty} \frac{x^2+\sqrt{2}}{3x^2-x}\right] = \ln \frac{1}{3}$

(f) $\lim_{x\to\infty} \frac{1-e^x}{2+8e^x} \cdot \frac{e^{-x}}{e^{-x}} = \lim_{x\to\infty} \frac{e^{-x}-1}{2e^{-x}+8} = \frac{-1}{8}$

Since $e^{-x} = \frac{1}{e^x} \to 0$ as $x\to\infty$.

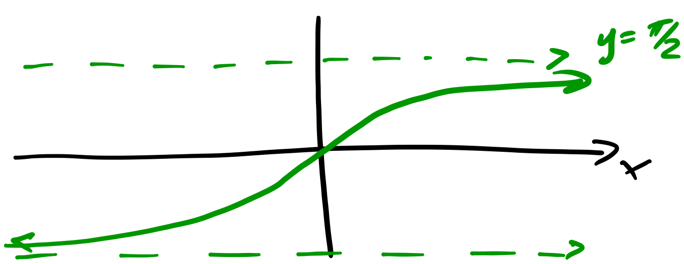
(g) $\lim_{x\to\infty} x^{-5/3}\cos x = \lim_{x\to} \frac{\cos x}{x^{5/3}} = 0$

$\frac{-1}{x^{5/3}} \le \frac{\cos x}{x^{5/3}} \le \frac{1}{x^{5/3}}$ for all x.

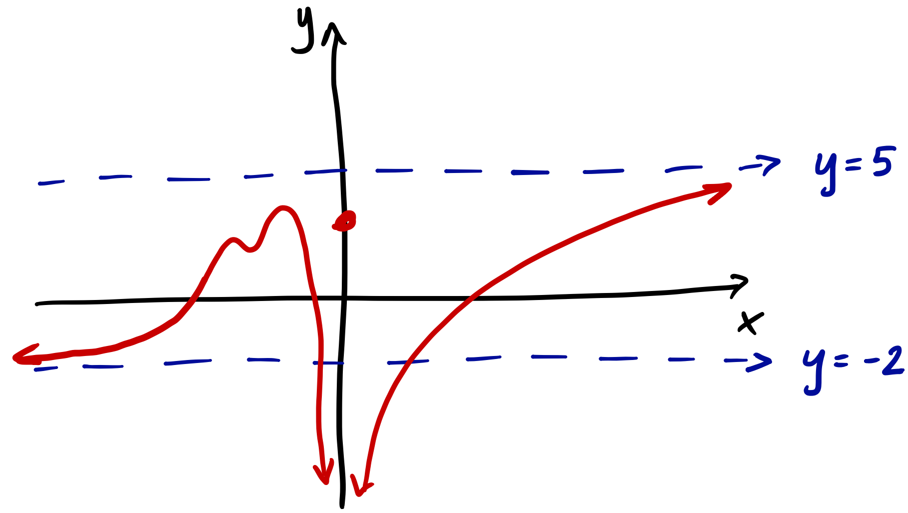
and $-1/x^{5/3} \to 0$ and $1/x^{5/3} \to 0$ as $x\to\infty$

(h) $\lim_{x\to-\infty} \arctan(2x) = -\pi/2$

from the graph.



2. Sketch the graph of an example of a function f that satisfies *all* of the given conditions:
(i) $\lim_{x\to 0} f(x) = -\infty$
(ii) $\lim_{x\to\infty} f(x) = 5$
(ii) $\lim_{x\to -\infty} f(x) = -2$



3. Let $v(t) = a(1 - e^{-gt/a})$ where a and g are fixed positive constants.

(a) Determine $\lim_{t\to\infty} v(t)$ and explain your reasoning.

$$\lim_{t\to\infty} a(1-e^{-gt/a}) = a$$

Reasoning: As $t\to\infty$, $-\frac{gt}{a} \to -\infty$. So $e^{-gt/a} \to 0$.

(b) Assume that $v(t)$ is the velocity of a falling raindrop and g is acceleration due to gravity. How would you interpret your answer to part (a)?

As time goes forward – that is, as the raindrop falls – its velocity approaches the number a which must be the terminal velocity of the raindrop